Transmission belt length and enclosing area

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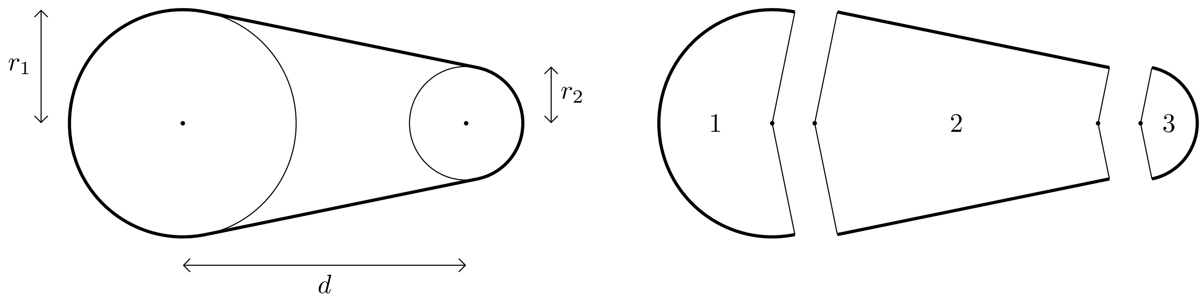
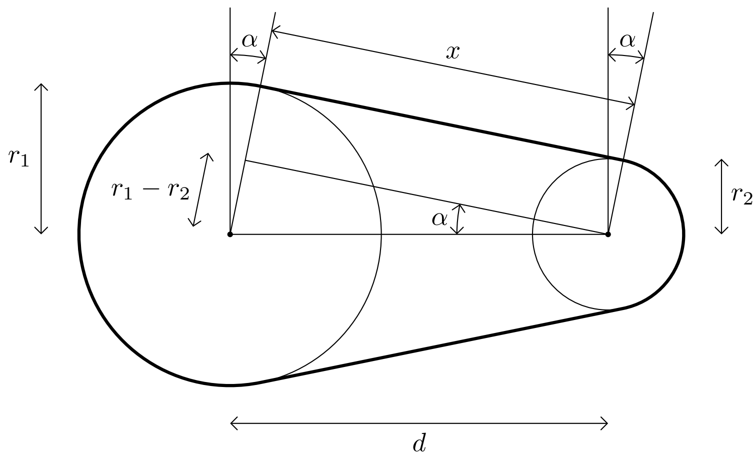
30 July 2016

In this article I will derive a formula for the length of a transmission belt and a formula for the area it encloses. The input variables are r_1 and r_2 for the radiuses of the circles and d for the distance between the centers of the two circles.

We split the shape into three parts. For each part we calculate the length of the bold line and the area.



In the drawing below we introduce the variables α and x. The angle α is in radians.



The right-angled triangle in the drawing lets us calculate α and x. To calculate x we can simply use the Pythagorean theorem. Part 1 and 3 are circle sectors of respectively $\pi + 2\alpha$ and $\pi - 2\alpha$ radians. Part 2 consists of rectangles and triangles and is not that hard to calculate.

$$\alpha = \arcsin\left(\frac{r_1 - r_2}{d}\right)$$

$$x = \sqrt{d^2 - (r_1 - r_2)^2}$$

$$L_1 = r_1(\pi + 2\alpha) \qquad A_1 = {r_1}^2(\pi/2 + \alpha)$$

$$L_2 = 2x \qquad A_2 = (r_1 + r_2)x$$

$$L_3 = r_2(\pi - 2\alpha) \qquad A_3 = {r_2}^2(\pi/2 - \alpha)$$

We sum up the parts to get the length L and the area A.

$$L = 2\arcsin\left(\frac{r_1 - r_2}{d}\right)(r_1 - r_2) + \pi(r_1 + r_2) + 2\sqrt{d^2 - (r_1 - r_2)^2}$$

$$A = \arcsin\left(\frac{r_1 - r_2}{d}\right)({r_1}^2 - {r_2}^2) + \frac{1}{2}\pi({r_1}^2 + {r_2}^2) + (r_1 + r_2)\sqrt{d^2 - (r_1 - r_2)^2}$$